3. A shopping mall has three automated teller machines (ATMs). Because the machines receive heavy use, they sometimes stop working and need to be repaired. Let the random variable X represent the number of ATMs that are working when the mall opens on a randomly selected day. The table shows the probability distribution of X.

Number of ATMs working when the mall opens	0	1	2	3
Probability	0.15	0.21	0.40	0.24

(a) What is the probability that at least one ATM is working when the mall opens?

(b) What is the expected value of the number of ATMs that are working when the mall opens?

(c) What is the probability that all three ATMs are working when the mall opens, given that at least one ATM is working?

(d) Given that at least one ATM is working when the mall opens, would the expected value of the number of ATMs that are working be less than, equal to, or greater than the expected value from part (b) ? Explain.

2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.
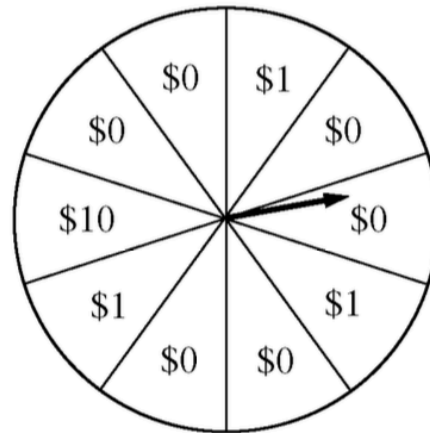
A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	−$8
$P(x)$			

(b) What is the expected value of the net contribution to the charity for one play of the game?

(c) The charity would like to receive a net contribution of $500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least $500 ?

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least $500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are $700 and $92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least $500 in 1,000 plays of the game.

2008 AP® STATISTICS FREE-RESPONSE QUESTIONS

3. A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

(a) Calculate the expected score for each player.

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is -1. List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of -1, and calculate the probability for each combination.

(c) Find the probability that the difference (Josephine minus Crystal) in their scores is -1.

(d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	-3	-2	-1	0	1	2
Probability	0.015			0.325	0.260	0.090

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

2. For an upcoming concert, each customer may purchase up to 3 child tickets and 3 adult tickets. Let C be the number of child tickets purchased by a single customer. The probability distribution of the number of child tickets purchased by a single customer is given in the table below.

c	0	1	2	3
$p(c)$	0.4	0.3	0.2	0.1

(a) Compute the mean and the standard deviation of C.

(b) Suppose the mean and the standard deviation for the number of adult tickets purchased by a single customer are 2 and 1.2, respectively. Assume that the numbers of child tickets and adult tickets purchased are independent random variables. Compute the mean and the standard deviation of the total number of adult and child tickets purchased by a single customer.

(c) Suppose each child ticket costs $15 and each adult ticket costs $25. Compute the mean and the standard deviation of the total amount spent per purchase.

2005 AP® STATISTICS FREE-RESPONSE QUESTIONS

2. Let the random variable X represent the number of telephone lines in use by the technical support center of a software manufacturer at noon each day. The probability distribution of X is shown in the table below.

x	0	1	2	3	4	5
$p(x)$	0.35	0.20	0.15	0.15	0.10	0.05

(a) Calculate the expected value (the mean) of X.

(b) Using past records, the staff at the technical support center randomly selected 20 days and found that an average of 1.25 telephone lines were in use at noon on those days. The staff proposes to select another random sample of 1,000 days and compute the average number of telephone lines that were in use at noon on those days. How do you expect the average from this new sample to compare to that of the first sample? Justify your response.

(c) The median of a random variable is defined as any value x such that $P(X \leq x) \geq 0.5$ and $P(X \geq x) \geq 0.5$. For the probability distribution shown in the table above, determine the median of X.

(d) In a sentence or two, comment on the relationship between the mean and the median relative to the shape of this distribution.
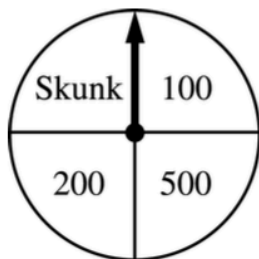
5. Contestants on a game show spin a wheel like the one shown in the figure above. Each of the four outcomes on this wheel is equally likely and outcomes are independent from one spin to the next.

   ▪ The contestant spins the wheel.

   ▪ If the result is a skunk, no money is won and the contestant's turn is finished.

   ▪ If the result is a number, the corresponding amount in dollars is won. The contestant can then stop with those winnings or can choose to spin again, and his or her turn continues.

   ▪ If the contestant spins again and the result is a skunk, all of the money earned on that turn is lost and the turn ends.

   ▪ The contestant may continue adding to his or her winnings until he or she chooses to stop or until a spin results in a skunk.

   (a) What is the probability that the result will be a number on all of the first three spins of the wheel?

   (b) Suppose a contestant has earned $800 on his or her first three spins and chooses to spin the wheel again. What is the expected value of his or her total winnings for the four spins?

   (c) A contestant who lost at this game alleges that the wheel is not fair. In order to check on the fairness of the wheel, the data in the table below were collected for 100 spins of this wheel.

Result	Skunk	$100	$200	$500
Frequency	33	21	20	26

   Based on these data, can you conclude that the four outcomes on this wheel are not equally likely? Give appropriate statistical evidence to support your answer.

2. Airlines routinely overbook flights because they expect a certain number of no-shows. An airline runs a 5 P.M. commuter flight from Washington, D.C., to New York City on a plane that holds 38 passengers. Past experience has shown that if 41 tickets are sold for the flight, then the probability distribution for the number who actually show up for the flight is as shown in the table below.

Number who actually show up	36	37	38	39	40	41
Probability	0.46	0.30	0.16	0.05	0.02	0.01

Assume that 41 tickets are sold for each flight.

(a) There are 38 passenger seats on the flight. What is the probability that all passengers who show up for this flight will get a seat?

(b) What is the expected number of no-shows for this flight?

(c) Given that not all passenger seats are filled on a flight, what is the probability that only 36 passengers showed up for the flight?

2001 AP® STATISTICS FREE-RESPONSE QUESTIONS

2. A department supervisor is considering purchasing one of two comparable photocopy machines, A or B. Machine A costs $10,000 and machine B costs $10,500. This department replaces photocopy machines every three years. The repair contract for machine A costs $50 per month and covers an unlimited number of repairs. The repair contract for machine B costs $200 per repair. Based on past performance, the distribution of the number of repairs needed over any one-year period for machine B is shown below.

Number of Repairs	0	1	2	3
Probability	0.50	0.25	0.15	0.10

You are asked to give a recommendation based on overall cost as to which machine, A or B, along with its repair contract, should be purchased. What would your recommendation be? Give a statistical justification to support your recommendation.

(a) In a certain manufacturing process, it is known that approximately 10% of the items produced are defective. A quality control scheme is set up, by selecting 20 items out of a large batch, and rejecting the whole batch if three or more are defective. Find the probability that the batch is rejected.

(b) Two boys, John and David, play a game with an unbiased die. The die will be thrown four times. David will give John £x if there is an odd number of '6's, otherwise John will give David £1. If the game is to be a fair one to both John and David, find x.

A multiple-choice test is taken by a large number of students. There are only 5 questions and 1 mark is given for the correct answer to a question. The resulting probability distribution is:

Marks (X)	1	2	3	4	5
Probability	0.02	0.08	0.26	0.41	0.23

Find the mean and standard deviation of this distribution.

If the marks are scaled up to go from 10 to 30, so that the new marks Y are given by the linear transformation

$$Y = 5X + 5$$

what will the mean and standard deviation of Y be?